5. INVERSION OF BOREHOLE RADAR DATA

5.1 Formulation

5.1.1 Forward modeling

The derivation of the inversion is commenced with the assumption that the following information on a target is known prior to borehole radar measurements:
- Material
- Dimension

Moreover, the followings must be known when boreholes are drilled.
- Direction
- Separation of the boreholes

Then an approximated first arrival time can be calculated for a model with assumptions that the surrounding medium is homogeneous, and the object is buried perpendicular to the plane between the boreholes.
In the case of a metallic pipe, it is assumed that the shortest propagation path gives a first arrival as shown in Fig. 5.1. In fact, the assumption is rough and must be different to an actual path. However, this model is used for the fast computation. The shapes of first arrival curves resulting from those paths are different with locations of the pipe, transmitter, and receiver. If a straight path crosses the pipe, the electromagnetic wave propagates along the perimeter of the pipe as a current flow because signals cannot pass through it. There are two possible paths of the resulting creeping waves on the pipe surface, $L$ and $L'$, as shown in Fig. 5.1. Since the inversion uses only the first arrival, the shorter one, $L$ in this case, is selected. The length of the path $L$ with the pipe at a particular location $(z,x)$ can be represented as a function of object location $(z,x)$, and the transmitter and receiver depth $z_t$ and $z_r$. Thus, the first arrival time can be given by

$$t_{cal}(z,x,z_t,z_r) = L(z,x,z_t,z_r)/v$$

(5.1)

where $v$ is the velocity of the electromagnetic wave in the subsurface medium.

**Fig. 5.1:** Geometry of cross-hole radar measurement for a metallic pipe. Straight ray and rays along the perimeter of the pipe are shown.
**Cavity**

A simple model with an air-filled cavity whose cross section is a circle in the inter-borehole plane is considered. Only Snell’s law is taken into account in order to decide for propagation paths as shown in Fig. 5.2. Same as the metallic pipe case, it is also rough assumption for the fast computation. When a ray travels through the cavity, the first arrival time can be expressed as

\[ t_{\text{cal}}(z, x, z_t, z_r) = \frac{(L_1 + L_2)}{v} + \frac{L_3}{c} \]  

(5.2)

where \( L_1 \) and \( L_2 \) are the propagation lengths in the surrounding medium, \( L_3 \) is that inside the cavity, \( c \) is the electromagnetic free space velocity in vacuum. Since the velocity in the air is higher than in the subsurface medium, it is possible that a first arrival given by a path passing through the cavity \( L_1 + L_2 + L_3 \) is faster than that by a straight path \( L' \) even though \( L_1 + L_2 + L_3 \geq L' \). Thus, faster arrival time must be selected.

**Fig. 5.2:** Geometry of cross-hole measurement for an air-filled cavity. Straight ray and ray passing through the cavity are shown.
5.1.2 Comparison of arrival time curves

Here two schemes are proposed to compare calculated and measured arrival curves. One is the gradient error scheme based on errors of the curve gradients, and the other one is the correlation scheme based on cross-correlation.

**Gradient error scheme**

It is easy to select a time \( t_{\text{meas}}(z_t, z_r) \) at the maximum amplitude in a measured trace for instance. The time is no longer true for the first arrival. It is, however, assumed that the curves connecting the times are parallel to the first arrival curve, meaning the medium has less anisotropy and dispersion, and the directivity effect of an antenna is small. This situation is illustrated in Fig. 5.3. The similarity between the measured maximum-amplitude curve \( t_{\text{meas}}(z_t, z_r) \) and the calculated first arrival curve \( t_{\text{cal}}(x, z_t, z_r) \) can be evaluated by taking an error \( e(z, x, z_r) \) of the curve gradients with respect to the receiver depth \( z_r \) and the integration.

\[
e(z, x, z_r) = \frac{1}{N_r} \int dz_r \left\| \partial_{z_r} t_{\text{meas}}(z_t, z_r) - \partial_{z_r} t_{\text{cal}}(x, z_t, z_r) \right\|
\]  

(5.3)

![Fig. 5.3: First arrival curve (broken line) and arrival curve picked with maximum amplitude (solid line) in a profile.](image)
where \( N_r \) is the number of receivers for a transmitter, and \( \| \cdot \|_n \) denotes \( L_n \) norm. This error can be calculated for each transmitter. Lower error indicates higher similarity between the two curves, meaning the higher probability of the object location. A total error for a series of all transmitter depths \( e_{\text{total}}(z, x) \) is defined by integrating \( e(z, x, z_i) \) along the transmitter depth \( z_i \) as

\[
e_{\text{total}}(z, x) = \frac{1}{N_t} \int \! dz_i \, e(z, x, z_i)
\]  

(5.4)

where \( N_t \) is the number of transmitters. In the map of \( e_{\text{total}}(z, x) \), a lower total error indicates a higher probability of the object location.

**Correlation scheme**

In this scheme, first arrival times are not required to be selected, but the measured data are directly used. To take a cross-correlation, a reference must be constructed from the calculated times \( t_{\text{cal}}(z, x, z_i, z_r) \); we construct a binary signal here

\[
H(z, x, z_i, z_r, t) = \delta(t - t_{\text{cal}}(z, x, z_i, z_r))
\]

(5.5)

where \( \delta(t) \) is dirac delta function. The one-dimensional cross-correlation between the reference \( H(z, x, z_i, z_r, t) \) and the measured data \( S(z, z_i, t) \) is calculated with respect to time \( t \), and the similarity for a transmitter \( R(z, x, z_i) \) is defined as the maximum absolute value of the integration of the cross-correlation along the receiver depth

\[
R(z, x, z_i) = \frac{1}{N_r} \max \left| \int \! d\tau \int \! dz \, S(z, z_i, \tau) H(z, x, z_i, z_r, t-\tau) \right|
\]

(5.6)

To obtain the total correlation \( R_{\text{total}}(z, x) \), those similarity values are integrated along the transmitter depth \( z_i \) as

\[
R_{\text{total}}(z, x) = \frac{1}{N_t} \int \! dz_i \, R(z, x, z_i)
\]

(5.7)

In the map of \( R_{\text{total}}(z, x) \), higher total similarity indicates higher probability of the object location.
5.1.3 Comparison criteria

In the gradient error scheme, the error between curve gradients is evaluated as expressed in Eq. (5.3). The most commonly employed norms are those based on the power. In the general case, the norms for Eq. (2.6) in Section 2.3 are given as follows.

\[
L_1 \text{ norm: } \| e \|_1 = \left[ \sum_i \left| d_{\text{meas}}^i - d_{\text{cal}}^i \right| \right]^{1/1} \tag{5.8a}
\]

\[
L_2 \text{ norm: } \| e \|_2 = \left[ \sum_i \left| d_{\text{meas}}^i - d_{\text{cal}}^i \right|^2 \right]^{1/2} \tag{5.8b}
\]

\[
L_n \text{ norm: } \| e \|_n = \left[ \sum_i \left| d_{\text{meas}}^i - d_{\text{cal}}^i \right|^n \right]^{1/n} \tag{5.8c}
\]

Successively higher norms give the large difference between \( d_{\text{meas}}^i \) and \( d_{\text{cal}}^i \) successively larger weight. The limiting case of \( n \to \infty \) gives nonzero weight to

![Fig. 5.4: Straight line fits to (z,d) pairs obtained by \( L_1 \), \( L_2 \), and \( L_\infty \) norm criteria. The \( L_1 \) norm gives least weight to the outlier (Menke, 1989).](image-url)
only the largest difference. It is equivalent to the selection of the largest difference with largest absolute value and is written as

$$L_{\infty} \text{ norm}: \|e\|_{\infty} = \max_{i} |d_{\text{meas}}^{i} - d_{\text{cal}}^{i}|.$$  \hspace{1cm} (5.8d)

An example how the norms differ is shown in Fig. 5.4. The $L_1$ norm, so called \textit{least-absolute-value criterion}, can fit a straight line as our expectation although an outlier exists, since it gives an equal weight to errors of different size. When a high-order norm is used, the line is more affected by the outlier, since it weights the larger errors preferentially. Thus, the $L_1$ norm is robust to outliers or noise, and it is denoted as \textit{robust estimation}. Inversion algorithms with an iterative approach generally use $L_2$ norm (\textit{least-square criterion}), because it is easy to calculate its derivative in order to derive the amount of the model update. As discussed in the former sections, the inversion employs the parametric approach. Claerbout and Muir (1973) gave a detailed discussion of the robustness of the $L_1$ norm criterion for the resolution of inverse problems. The gradient scheme, therefore, uses the criterion, and Eq. (5.3) can be rewritten as

$$e(z, x, z_r) = \frac{1}{N_r} \int dz_r \left| \partial_{z_r} t_{\text{meas}}(z_r, z_r) - \partial_{z_r} t_{\text{cal}}(z, x, z, z_r) \right|.$$  \hspace{1cm} (5.9)
5.2 Examples with Synthetic Data

The effectiveness of the inversion schemes is demonstrated with synthetic data generated by the three-dimensional finite difference time domain (FDTD) method.

5.2.1 Metallic pipe in homogeneous medium

The FDTD model for a metallic pipe in a homogeneous medium is constructed as Fig. 5.5. The medium has a constant permittivity of $20\varepsilon_0$ and relatively high conductivity of 0.01 S/m, simulating a moist soil, which is the typical medium beneath roadways. The diameter of the pipe is 1 m, and its location is at a depth of 12 m and 2 m horizontally distant from the transmitting borehole. Dipole antennas are modeled as a transmitting and receiving antennas in boreholes with the separation of 3.3 m. A second derivative of a Gaussian pulse is employed as the source excitation. The frequency range is 10-200 MHz, which gives a wavelength of

![Fig. 5.5: An $x$-$z$ cross-section of the model at $y = 5$ m for generating synthetic radar profiles in the FDTD simulation. The metallic pipe having a diameter of 1 m is located in a homogeneous medium at a depth of 12 m and 2 m horizontally away from the transmitting borehole.](image-url)
1-2 m in the medium. Other parameters are listed in Table 5.1.

Fig. 5.6 shows raw radar profiles obtained by the simulation. Although we may be able to estimate the vertical location, the horizontal location is almost impossible to be estimated directly from the profiles. As summarized in Table 5.2, times at positive maximum amplitudes are selected from measured data for the gradient error scheme, and a rectangular pulse having a width of one pixel (i.e., one pixel is set to 1 and otherwise 0 in a trace) is employed as a reference signal for the correlation scheme. Fig. 5.7 shows the selected curves and the calculated one with the exact location of the modeled pipe. We can observe that the curves are almost parallel to each other even so the rough forward modeling is used. Fig. 5.8 shows the distributions of errors $e(z,x,z_t)$ for a transmitter depth calculated from Eq. (5.9). The low error regions are not localized and are spreading along the lines connecting the transmitter and the pipe. These figures do not clearly indicate the pipe location. Fig. 5.9 shows the total error distribution $e_{total}(z,x)$ obtained by Eq. (5.4). It has a localized low error region, and the minimum error is located at a depth of 12 m and a horizontal distance of 2 m away from the transmitting borehole. The lowest error indicates the exact location of the pipe. Fig. 5.10 shows the maps of the similarity $R(z,x,z_t)$ in the correlation scheme defined in Eq. (5.6). Similar to the case of the gradient error scheme, the high value regions are spreading. Fig. 5.11 shows the total similarity $R_{total}(z,x)$. The high similarity region is localized, and the maximum value indicates a depth of 12 m and 2 m from the transmitting borehole. It also indicates the exact location of the pipe.

Table 5.1: Parameters for the FDTD calculation with the metallic pipe.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell size</td>
<td>$0.1 \text{ m} \times 0.1 \text{ m} \times 0.1 \text{ m}$</td>
</tr>
<tr>
<td>Number of cells</td>
<td>$100 \times 100 \times 80$</td>
</tr>
<tr>
<td>Time step</td>
<td>0.19258078 ns</td>
</tr>
<tr>
<td>Number of time steps</td>
<td>1500</td>
</tr>
<tr>
<td>Boundary condition</td>
<td>10 layers PML</td>
</tr>
<tr>
<td>Excitation pulse</td>
<td>2nd derivative Gaussian pulse</td>
</tr>
<tr>
<td>Transmitter depths</td>
<td>11-13 m, 0.5 m step</td>
</tr>
<tr>
<td>Receiver depths</td>
<td>10-14 m, 0.1 m step</td>
</tr>
</tbody>
</table>
Table 5.2: Parameters for inverting the synthetic data with the metallic pipe in a homogeneous medium.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Gradient error scheme</th>
<th>Correlation scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Used Tx depths</td>
<td>All depths</td>
<td></td>
</tr>
<tr>
<td>Used Rx depths</td>
<td>All depths</td>
<td></td>
</tr>
<tr>
<td>Assumed location interval</td>
<td>0.1 m in depth, 0.1 m in distance</td>
<td></td>
</tr>
<tr>
<td>Assumed permittivity</td>
<td>$20\varepsilon_0$</td>
<td></td>
</tr>
<tr>
<td>Arrival time picking</td>
<td>Times at max. positive amp.</td>
<td>-</td>
</tr>
<tr>
<td>Reference signal</td>
<td>-</td>
<td>1 pixel rectangular pulse</td>
</tr>
</tbody>
</table>
Fig. 5.6: Synthetic radar profiles generated for the model shown in Fig. 5.5. The transmitter is set at depths of 11.0 m (a), 11.5 m (b), 12.0 m (c), 12.5 m (d), and 13.0 m (e).
Fig. 5.7: Arrival curve selected by maximum amplitude (black solid line) and first arrival curve (white broken line). The transmitter depths are 11.0 m (a), 11.5 m (b), 12.0 m (c), 12.5 m (d), and 13.0 m (e).
Fig. 5.8: Distributions of error $e(z,x,z_0)$ for transmitters at depths of 11.0 m (a), 11.5 m (b), 12.0 m (c), 12.5 m (d), and 13.0 m (e), respectively. Lower error indicates higher probability of the pipe location.
Fig. 5.9: Distribution of the total error \( e_{\text{total}}(z,x) \). The cross indicating the location of the lowest error is at a depth of 12 m and 2 m away from the transmitting borehole.
Fig. 5.10: Distributions of similarity $R(z,x,z_\text{r})$ for transmitters at depths of 11.0 m (a), 11.5 m (b), 12.0 m (c), 12.5 m (d), and 13.0 m (e). Higher similarity indicates higher probability of the pipe location.
Fig. 5.11: Distribution of the total similarity $R_{\text{total}}(z,x)$. The cross indicating the location of the highest similarity is at a depth of 12 m and 2 m away from the transmitting borehole.
5.2 Examples with Synthetic Data

5.2.2 Metallic pipe in inhomogeneous medium

For demonstrating a more realistic situation, an inhomogeneous medium is simulated in a three-dimensional space by means of a stochastic fractal model. It is known that fluctuations of subsurface properties are generally scale invariant or fractal. The stochastic fractal approach is appropriate and realistic to model distributions of electric properties in a subsurface medium. Such fluctuations can be simulated by taking the square root of the desired power spectrum, randomizing the phase, and taking the inverse Fourier transform (Lampe and Holliger, 2003; Lampe, 2003). The following power spectrum is used in this case.

\[ P(k) \propto \frac{1}{(1 + k^2 a^2)^{\nu + \frac{D}{2}}} , \]  

(5.10)

where \( k \) is the spatial wavenumber, \( a \) is the correlation length, \( \nu \) is the Hurst number, and \( E \) is the Euclidean dimension. The fractal dimension \( D \) of the corresponding stochastic process is given by

\[ D = E + 1 - \nu . \]  

(5.11)

We choose \( \nu = 0.1 \), and \( a = 5.0 \) for the calculation space shown in Fig. 5.5, yielding \( E = 3 \), and \( D = 3.9 \). From the fractal fluctuations, electric properties can be simulated as

\[ \begin{align*}
\end{align*} \]

Fig. 5.12: Inhomogeneous background medium in the FDTD simulation. The cross section at \( y = 5 \) m (a), and the histogram of the permittivities showing a Gaussian distribution with the mean value of about \( 20 \epsilon_0 \) and the standard deviation of \( 1.1 \epsilon_0 \) (b).
where \( \mathbf{r} \) denotes a specific location, and \( s_0 \) and \( \Delta s \) are the deterministic background value and stochastic component of a model parameter, respectively. In this case, the permittivity is subdivided into 14 permittivity values ranging from \( 16.5 \varepsilon_0 \) to \( 23.0 \varepsilon_0 \), and the conductivity is considered to be constant as well as in the homogeneous case. Fig. 5.12(a) shows the cross section of the obtained inhomogeneous medium model at \( y = 5 \text{ m} \), and Fig. 5.12(b) shows the histogram of the distribution. In this model, the mean is about \( 20 \varepsilon_0 \), and the standard deviation is \( 1.1 \varepsilon_0 \).

The inversion is performed with the same parameters as for the homogeneous medium case, thus the variations of the permittivity are not taken into account, and a constant value of \( 20 \varepsilon_0 \) is used in the calculation. Fig. 5.13 shows the resultant probability distributions. Although the localized regions are deformed comparing to the homogeneous medium case shown in Figs. 5.9 and 5.11, they show the minimum error and maximum similarity to be at the exact location.

The fact that the inversion works well even for an inhomogeneous medium case assuming a constant permittivity indicates that the technique is robust to inhomogeneities. It is a great advantage for the practical use because we can easily specify the parameters and do not have to consider it in more detail. Therefore, it can be used by non-specialists of radar/geology with little experience and poor knowledge. Conversely, the technique is not sensitive to small variations of the medium property, thus it is not suitable for an estimation of a subsurface structure in detail.
5.2 Examples with Synthetic Data

Fig. 5.13: Total error (a) and similarity (b) maps derived from the inversion of synthetic data for the inhomogeneous medium using the gradient error and correlation schemes, respectively.
5.2.3 Air-filled cavity

The synthetic data for an air-filled cavity is constructed by FDTD with the model shown in Fig. 5.14. A constant permittivity of $5 \varepsilon_0$ and conductivity $0.005 \ \text{S/m}$ is chosen for a medium representing granite. The cavity having the diameter of 3 m and round shape is modeled at a depth of 80 m and 5 m distant from the receiving borehole. The borehole separation is 19 m. Other parameters are listed in Table 5.3.

Fig. 5.15 shows raw radar profiles. Solid and broken lines show the measured maximum-amplitude and calculated first arrival curves, respectively. The vertical location of the target can easily be estimated to about 80 m. Parameters for the inversion are summarized in Table 5.4. The distributions of errors $e(z, x, z_t)$ are shown in Fig. 5.16. The low error region is localized near the modeled location in the distributions for the transmitter at 75, 80, and 85 m. Fig. 5.17 shows the total error $e_{\text{total}}(z, x)$. The lowest error is at a depth of 80 m and 5 m away from the

![Fig. 5.14: An x-z cross-section of the model at y=5 m for generating synthetic radar profiles in the FDTD simulation. The cavity having a diameter of 3 m is located at a depth of 80 m and 5 m away from the receiving borehole.](image)
receiving borehole, clearly showing the exact location. The distributions of the similarity $R(z,x,z_i)$ and the total similarity $R_{total}(z,x)$ by the correlation scheme are shown in Fig. 5.18 and 5.19, respectively. In the distribution, the highest similarity is not at the correct location. The reason for the inaccurate result to occur is that the electromagnetic wave can pass through the air-filled cavity, and there are many propagation paths. Thus, measured data can have relatively high energy at not only the first arrivals and scattered wavefields. Since the scheme directly takes the cross-correlation between the binary reference and raw measured data, the correlation shows high responses at not only the first arrivals but also the scattered wavefields. The gradient error scheme is suitable for cavity localization, whereas the correlation scheme is not.

Table 5.3: Parameters for the FDTD calculation with the air-filled cavity.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell size</td>
<td>$0.1\text{m} \times 0.1 \text{m} \times 0.1 \text{m}$</td>
</tr>
<tr>
<td>Number of cells</td>
<td>$250 \times 250 \times 80$</td>
</tr>
<tr>
<td>Time step</td>
<td>$0.19258078 \text{ns}$</td>
</tr>
<tr>
<td>Number of time steps</td>
<td>1600</td>
</tr>
<tr>
<td>Boundary condition</td>
<td>10 layers PML</td>
</tr>
<tr>
<td>Excitation pulse</td>
<td>2nd derivative Gaussian pulse</td>
</tr>
<tr>
<td>Transmitter depths</td>
<td>70-90 m, 5 m step</td>
</tr>
<tr>
<td>Receiver depths</td>
<td>70-90 m, 0.2 m step</td>
</tr>
</tbody>
</table>

1 This fact is following the principle of Fermat, known also as the principle of the shortest optical path. The fast arrival is given by the shortest propagation path, but the energy may follow a different one.
Fig. 5.15: Synthetic radar profiles generated for the model shown in Fig. 5.14. Arrival curve selected by maximum amplitude (solid line) and first arrival curve (broken lines) are shown. The transmitter is set at depths of 70 m (a), 75 m (b), 80 m (c), 85 m (d), and 90 m (e).
5.2 Examples with Synthetic Data

Fig. 5.16: Distributions of error \( e(z, x, z_i) \) for transmitters at depths of 70 m (a), 75 m (b), 80 m (c), 85 m (d), and 80 m (e), respectively.
5. Inversion of Borehole Radar Data

**Fig. 5.17:** Distribution of the total error $e_{\text{total}}(z,x)$. The cross indicating the location of the lowest error is at a depth of 80 m and 5 m away from the receiving borehole.

**Table 5.4:** Parameters for inverting the synthetic data with the air-filled cavity.

<table>
<thead>
<tr>
<th></th>
<th>Gradient error scheme</th>
<th>Correlation scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Used Tx depths</td>
<td>All depths</td>
<td></td>
</tr>
<tr>
<td>Used Rx depths</td>
<td>All depths</td>
<td></td>
</tr>
<tr>
<td>Assumed location interval</td>
<td>0.5 m in depth, 0.5 m in distance</td>
<td></td>
</tr>
<tr>
<td>Assumed permittivity</td>
<td>$5 \varepsilon_0$</td>
<td></td>
</tr>
<tr>
<td>Arrival time picking</td>
<td>Times at max. amplitude</td>
<td>-</td>
</tr>
<tr>
<td>Reference signal</td>
<td>-</td>
<td>1 pixel rectangular pulse</td>
</tr>
</tbody>
</table>
5.2 Examples with Synthetic Data

Fig. 5.18: Distributions of similarity $R(z, x, z')$ for transmitters at depths of 70 m (a), 75 m (b), 80 m (c), 85 m (d), and 80 m (e), respectively.
Fig. 5.19: Distribution of the total similarity $R_{\text{total}}(z,x)$. The cross indicates the location of the highest similarity.
5.3 Experimental Results

5.3.1 Radar system

The borehole radar system used in field measurements has been developed by the Division of Environmental and Resources Survey, Center for Northeast Asian Studies (CNEAS) at the Tohoku University. It is a vector network analyzer based stepped-frequency radar system, and the system diagram is shown in Fig. 5.20. An analog optical link system is employed in this system to electrically isolate the antennas from the surface units of the system and to achieve an ideal antenna radiation pattern. In the measurements, the old system type (Miwa et al., 1999) was used for a cavity example, while the new type (Takahashi et al., 2002) was employed for a metallic pipe.

Fig. 5.20: Borehole radar measurement system. It is controlled by vector network analyzer with an optical fiber connection.
5.3.2 Metallic pipe case #1

The first example of field measurements for a metallic pipe was carried out in an urban area in the city of Sendai, Japan in September 2005. The objective of this measurement is to locate a metallic pipe for water supply which needs to be renovated because of designing and installing a new subway station. The pipe has a diameter of 0.9 m, and it is buried at a depth of approximately 12 m. Two boreholes were drilled with a separation of 3.3 m on both sides of the pipe as shown in Fig. 5.21. We carried out various borehole radar surveys such as single-hole, cross-hole parallel, and cross-hole fan measurements. Dipole antennas having a length of 900 mm were used as transmitter and receiver, respectively, in these measurements.

**Rough localization by single-hole measurement**

The distance between the transmitter and receiver was set to 1.0 m for the single-hole measurement. The raw radar profiles are shown in Fig. 5.22. A hyperbolic curve can be found at a depth of about 12 m and time of 40 ns in Fig. 5.22(b), whereas we cannot find it in (a). The reflection may be attenuated and cannot be received in the measurement from borehole BH1. It indicates that borehole BH2 is closer to the pipe than borehole BH1.

![Fig. 5.21: Geometrical sketch of the site for field measurements in Sendai, Japan, 2004.](image)
Permittivity estimation by cross-hole parallel measurement

In the cross-hole parallel measurement, the transmitter and receiver were installed into boreholes BH1 and BH2, respectively. The acquired profile is shown in Fig. 5.23. To estimate permittivity of the medium for the inversion, first arrival times are selected from the traces by a threshold at 0.2 % of the maximum amplitudes. The estimated permittivity is shown in Fig. 5.24. Due to strong attenuation as shown in the profile, permittivity cannot be defined at depths of 10.0-11.2 m. The permittivity increases at a depth of 12.5 m. It is caused by an interface of layers. In general, an installation of a pipe is conducted as follows: a trench is dug, the pipe is laid, and then it is covered with other types of soil or gravel. This means that there are two types of media around the buried pipe. In this case, strong scattering may occur at the boundary as shown in the profiles and the estimated permittivity. From the result, we estimate that the permittivity of the medium above the pipe was $20 \varepsilon_0$.

![Fig. 5.22: Radar profiles measured by the single-hole measurements in boreholes BH1 (a) and BH2 (b).](image-url)
Fig. 5.23: Radar profile measured by cross-hole parallel measurements. The transmitter and receiver are scanned in boreholes BH1 and BH2, respectively.

Fig. 5.24: Permittivity distribution estimated from cross-hole parallel measurements. Permittivity around the boreholes is presumed to $20\varepsilon_0$.

**Inversion of cross-hole fan data**

According to the fact that borehole BH2 is closer to the pipe than borehole BH1, the transmitter was set in BH1 and the receiver was scanned in BH2. Because the change of the arrival curve by an object is more remarkable in case the object is closer to a receiver as was demonstrated by Zhou and Sato (2004). The installation depths of the transmitter were 11.0, 11.5, 12.0, 12.5, and 13.0 m. The receiver was scanned from 10.0 to 14.0 m in depth, and the interval was 0.1 m. The measured
profiles are shown in Fig. 5.25.

In the inversion, noisy signals and signals affected by inhomogeneities and geological discontinuities must be excluded because of the assumptions of the homogeneous medium and the simplified propagation path in the forward modeling. Times at maximum amplitudes are selected from measured data for the gradient error scheme, and three pixel rectangular pulses are used as the reference signals for the correlation scheme as summarized in Table 5.5. The moving average for three traces is taken in order to obtain smooth curves. The pipe locations are chosen with 0.1 m steps in both schemes. The inversion results shown in Fig. 5.26 indicate similar locations, a depth of 12.1 m and 2.1 m from BH1 by the gradient error scheme, and a depth of 12.0 m and 2.1 m from BH1 by the correlation scheme. These locations agree well with the information estimated by the single-hole measurements.

Maps do not indicate the pipe location as clearly as in the case of synthetic data. This is partly caused by the transmitter positions. In the inversion of synthetic data, data, acquired at various depths especially both above and below the pipe, are necessary to obtain a well-localized image. In this case, the data acquired by the transmitter below the pipe cannot be used due to the strong geological effects, as we can observe in Figs. 5.25(d) and (e). It may be possible to take into account this situation by using a two-layer model in the forward modeling. To keep the modeling simple, here we use a homogeneous model by excluding some traces. One can easily know the location from the results, although the data are few and sparse; and conversely, it would be possible to locate the pipe with only two data sets acquired by the transmitter arrangements at positions above and below the target.

In this case, we excluded the data affected by the geology. This situation is simulated by using synthetic data. Fig. 5.27 shows the result. Compared with Figs. 5.9 and 5.11, the localized regions are deformed, and the estimated locations are slightly changed, whereas the change is only 0.1 m in horizontal direction in the gradient error scheme and in depth in the correlation scheme. From the inversion result, the actual resolution may be estimated as about 15 cm. It represents a permissible error for this kind of non-destructive investigations and provides sufficiently high resolution considering the object dimension.
Fig. 5.25: Radar profiles measured by cross-hole fan measurements. The transmitter is set in borehole BH1 at depths of 11.0 m (a), 11.5 m (b), 12.0 m (c), 12.5 m (d), and 13.0 m (e). The receiver is scanned in borehole BH2.
**Traveltime tomography with cross-hole fan data**

Fig. 5.28 shows a result of straight-ray tomography obtained from the same cross-hole data. It is very noisy and almost impossible to discern anything from the tomogram. This insufficient result is due to the sparse sampling and narrow angular coverage. In addition, ray tomography generally works well for objects whose properties are gradually changed, but it cannot clearly image objects which possess high contrast of properties to the medium, such as an artificial object like a metallic pipe. On the other hand, the inversion can work better for such high constant objects than low contrast ones because the deformation of the arrival time curve is greater for a high contrast object. Therefore, the inversion method is more suitable to locating a metallic pipe than the tomographic technique.

<table>
<thead>
<tr>
<th>Table 5.5: Parameters for inverting the data measured in Sendai, Japan, 2004.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Used Tx depths</td>
</tr>
<tr>
<td>Used Rx depths</td>
</tr>
<tr>
<td>Assumed location interval</td>
</tr>
<tr>
<td>Assumed permittivity</td>
</tr>
<tr>
<td>Arrival time picking</td>
</tr>
<tr>
<td>Reference signal</td>
</tr>
</tbody>
</table>
Fig. 5.26: Total error (a) and similarity (b) maps derived from the inversion of field data using the gradient error and correlation schemes, respectively. The minimum error denoted by the white cross appears at a depth of 12.1 m and 2.1 m away from borehole BH1, while the maximum similarity indicated by the black cross occurs at a depth of 12.0 m and 2.2 m away from borehole BH1.
Fig. 5.27: Total error (a) and similarity (b) maps derived from the inversion of synthetic data excluding some traces using the gradient error and correlation schemes, respectively. The minimum error denoted by the white cross appears at a depth of 12.0 m and 2.1 m away from the transmitter borehole, while the maximum similarity indicated by the black cross occurs at a depth of 11.9 m and 2.0 m away from the transmitter borehole.
**Fig. 5.28:** Velocity map obtained by traveltime tomography considering straight rays. There is no clear contrast in this image, and it is almost impossible to recover any valid information.
5.3.3 Metallic pipe case #2

The second case of field experiments for metallic pipes was carried out in the city of Amagasaki, Japan in August 2005. In the test site, six ductile iron pipes, having a diameter of 0.5 m, are buried; three of them are buried at a depth of 1.25 m, and the others are at 2.25 m with slightly bended connections as shown in Fig. 5.29. The host rock is a sedimentary rock, and the pipes are covered with sand. Boreholes were drilled with separations of about 1, 2, and 3 m at both sides of the pipes. Since there is no groundwater, we poured water in the wells when the radar system was installed. Dipole antennas whose length is 500 mm were used as the transmitter and receiver. We carried out cross-hole parallel, fan, and vertical radar profiling (VRP) measurements with our borehole radar and the one-dimensional surface GPR surveys with RAMAC/GPR™.

![Diagram of test site](image_url)

**Fig. 5.29:** Top view of the test site in Amagasaki, Japan, 2005.

---

1. RAMAC/GPR™ is registered a trademark of MALÅ GeoScience.
**Estimation of layered structure by cross-hole parallel measurements**

A layered structure in this site is firstly checked by investing the attenuation. Since the actual transmitting waveform cannot be estimated easily, the exact attenuation can hardly be estimated. Here, instead, the relative attenuation is calculated from cross-hole parallel data. From acquired spectra, relative attenuation \( \alpha_r \) is calculated as

\[
\alpha_r(z, \omega) = \frac{\tilde{E}(z, \omega)}{\min_z [\tilde{E}(z, \omega)]}
\]  

where \( \tilde{E}(z, \omega) \) is the received power at a depth \( z \). Fig. 5.30 shows cross-sections of relative attenuation derived from data measured between boreholes BH03 and BH25, BH07 and BH06, and BH11 and BH10, which include pipes at 2.25 m depth, respectively. From these figures, we can divide a depth section into three regions. The attenuation is very high at depths of 2.0 - 2.5 m mainly due to the buried pipe. Above the depth range, attenuation is relatively high, while it is low beneath the pipe. This result clearly displays the layered structure for which the host medium has relatively low attenuation, and the sand covering the pipe has relatively high attenuation.

![Fig. 5.30: Vertical distributions of relative attenuation constants between BH03 and BH25 (a), between BH07 and BH06 (b), and between BH11 and BH10 (c). The solid, dotted and broken-dotted lines indicate the attenuations at 40, 60, and 80 MHz, respectively.](image-url)
Estimation of permittivity distribution by VRP

The permittivity distribution is roughly estimated from VRP data. The obtained profiles, whose transmitters were in boreholes BH01 and BH02 and for which the receiver was scanned perpendicular to the pipe, are shown in Figs. 5.31 and 5.32. First arrival times are simply selected by thresholding, which is defined as 2% of maximum amplitudes. By assuming straight ray paths, the propagation velocity and permittivity can be calculated from the arrival times and antenna positions. Then the velocity or permittivity is assigned to grids on which the assumed ray path, and lower velocities or higher permittivities are overwritten\(^3\). Fig. 5.33 shows the results for the permittivity from each borehole, and Fig. 5.34 shows the combined one. We can observe that high permittivities are distributed near the boreholes and in deep regions. From the distribution, the permittivity is estimated to be \(9 \varepsilon_0\) for the host medium, and \(5 \varepsilon_0\) for the covered sand.

---

\(^3\) In this case, the target is a metallic pipe, which is shown as high permittivity. When the target has low permittivity, lower permittivities should be overwritten.
Fig. 5.31: Radar profiles obtained by VRP measurements. The transmitter is scanned in borehole BH01, and the receiver is set on the ground surface at 0.5 m (a), 1.0 m (b), 1.5 m (c), 2.0 m (d), 2.5 m (e), and 3.0 m (f) away from borehole BH01.
5.3 Experimental Results

Fig. 5.32: Radar profiles obtained by VRP measurements. The transmitter is scanned in borehole BH02, and the receiver is set on the ground surface at 0.5 m (a), 1.0 m (b), 1.5 m (c), 2.0 m (d), 2.5 m (e), and 3.0 m (f) from borehole BH01.
Fig. 5.33: Permittivity distribution obtained from VRP data at boreholes BH01 (a) and BH02 (b).
5.3 Experimental Results

Fig. 5.34: Permittivity distribution between boreholes BH01 and BH02 derived from VRP data.

Inversion of cross-hole fan data

Cross-hole fan measurements were carried out for the pipes at 2.25 m in depth using the six boreholes. Some of the radar profiles are shown in Figs. 5.35-40. Parameters for the measurements and inversion are summarized in Tables 5.6 and 5.7. The gradient error scheme is used for all of the inversions in order to obtain Fig. 5.41. The resultant locations are summarized in Table 5.8. The results from the data at boreholes BH25 - BH03, BH03 - BH25, BH10 - BH11, and BH11 - BH10, respectively, are in good agreement with the true locations. For boreholes BH06 and BH07, however, no solutions are obtained. It is due to the particular locations of the boreholes close to the trench dug for the pipe installation. The trench produces the interface of two different mediums both horizontally and vertically. As the vertical profile indicates for surface GPR measurements shown in Fig. 5.42 and for a picture of the pipe installation shown in Fig. 5.43, the width of the trench seems to be about 2 m. The borehole separation between BH06 and BH07 is also 2 m. Since the wells are very close to the interface, scattering on the boundary interferes with the first arrivals. The complex scattered first arrivals can be seen in the profiles. For BH25 - BH03 and BH10 - BH11 (Figs. 5.35, 5.36, 5.39, and 5.40), the first arrivals show one lines, whereas those for BH06 - BH07 (Figs. 5.37, and 5.38) seem forked. The
data for BH25 - BH03 and BH10 - BH11 are of course interfered by scattering, however their first arrivals are not disturbed. Since these boreholes are farther from the boundary, it can be assumed to arrive later.

As described in Table 5.8, the estimation accuracy is about 10 cm. It is sufficient if the measurement conditions are considered. The diameter of the drilled well is 10 cm and the down-hole sonde of the radar is 4 cm, thus it has 6 cm spacing.

**Traveltime tomography with cross-hole fan data**

Fig. 5.44 shows results of straight-ray tomography obtained from the same cross-hole data. The results from the data for BH25 - BH03 and BH03 - BH25 are very noisy and it is almost impossible to locate the pipe from the tomogram. For the data of BH06 - BH07, BH07 - BH06, and BH11 - BH10, the tomograms show two different velocity regions, shallow and deep. Nevertheless, the images are not clear and it is very difficult to locate the pipe. The calculation for the data BH10 - BH11 is divergent and the results cannot be obtained.

### Table 5.6: Parameters for cross-hole fan measurements in Amagasaki, Japan, 2005.

<table>
<thead>
<tr>
<th>Tx borehole</th>
<th>Rx borehole</th>
<th>Separation</th>
<th>Tx depths</th>
<th>Rx depths</th>
</tr>
</thead>
<tbody>
<tr>
<td>BH25</td>
<td>BH03</td>
<td>3.014 m</td>
<td>0.5-3.5 m, 0.5 m step</td>
<td>0.5-3.5 m, 0.1 m step</td>
</tr>
<tr>
<td>BH03</td>
<td>BH25</td>
<td>1.924 m</td>
<td>0.5-2.5 m, 0.5 m step</td>
<td>0.5-3.5 m, 0.1 m step</td>
</tr>
<tr>
<td>BH06</td>
<td>BH07</td>
<td>1.031 m</td>
<td>0.5-2.5 m, 0.5 m step</td>
<td>0.5-3.5 m, 0.1 m step</td>
</tr>
</tbody>
</table>
5.3 Experimental Results

**Fig. 5.35:** Radar profiles obtained by cross-hole fan measurements for the transmitter in borehole BH25 at depths of 1.0 m (a), 2.0 m (b), and 3.0 m (c), respectively. The receiver is inserted in borehole BH03.

**Fig. 5.36:** Radar profiles obtained by cross-hole fan measurements for the transmitter in borehole BH03 at depths of 1.0 m (a), 2.0 m (b), and 3.0 m (c), respectively. The receiver is inserted in borehole BH25.

**Fig. 5.37:** Radar profiles obtained by cross-hole fan measurements for the transmitter in borehole BH06 at depths of 1.0 m (a), 2.0 m (b), and 3.0 m (c), respectively. The receiver is inserted in borehole BH07.
Fig. 5.38: Radar profiles obtained by cross-hole fan measurements for the transmitter in borehole BH07 at depths of 1.0 m (a), 2.0 m (b), and 3.0 m (c), respectively. The receiver is inserted in borehole BH06.

Fig. 5.39: Radar profiles obtained by cross-hole fan measurements for the transmitter in borehole BH10 at depths of 1.0 m (a), 2.0 m (b), and 3.0 m (c), respectively. The receiver is inserted in borehole BH11.

Fig. 5.40: Radar profiles obtained by cross-hole fan measurements for the transmitter in borehole BH11 at depths of 1.0 m (a), 2.0 m (b), and 3.0 m (c), respectively. The receiver is inserted in borehole BH10.
### Table 5.7: Parameters for inverting the data measured in Amagasaki, Japan, 2005.

<table>
<thead>
<tr>
<th>Tx borehole</th>
<th>Rx borehole</th>
<th>Assumed permittivity</th>
<th>Tx depths</th>
<th>Rx depths</th>
<th>Arrival time picking</th>
</tr>
</thead>
<tbody>
<tr>
<td>BH25</td>
<td>BH03</td>
<td>$5 \varepsilon_0$</td>
<td>0.5-3.5 m,</td>
<td>0.5-3.5 m,</td>
<td>Thresholding</td>
</tr>
<tr>
<td>BH03</td>
<td>BH25</td>
<td></td>
<td>0.5 m step,</td>
<td>0.1 m step,</td>
<td>(40 % of max. amp)</td>
</tr>
<tr>
<td>BH06</td>
<td>BH07</td>
<td>$5 \varepsilon_0$</td>
<td>0.5-2.5 m,</td>
<td>1.0-2.5 m,</td>
<td>Thresholding</td>
</tr>
<tr>
<td>BH07</td>
<td>BH06</td>
<td></td>
<td>0.5 m step,</td>
<td>0.1 m step,</td>
<td>(10 % of max. amp)</td>
</tr>
<tr>
<td>BH10</td>
<td>BH10</td>
<td>$5 \varepsilon_0$</td>
<td>0.5-2.5 m,</td>
<td>1.0-2.4 m,</td>
<td>Thresholding</td>
</tr>
<tr>
<td>BH11</td>
<td>BH11</td>
<td></td>
<td>0.5 m step,</td>
<td>0.1 m step,</td>
<td>(45 % of max. amp)</td>
</tr>
</tbody>
</table>

### Table 5.8: Estimated locations of the buried pipes.

<table>
<thead>
<tr>
<th>Tx borehole</th>
<th>Rx borehole</th>
<th>Estimated location [m]</th>
<th>True location [m]</th>
<th>Error [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>H  V</td>
<td>H  V</td>
<td></td>
</tr>
<tr>
<td>BH25</td>
<td>BH03</td>
<td>1.65 2.25</td>
<td>1.646 2.25</td>
<td>0.004 0.00</td>
</tr>
<tr>
<td>BH03</td>
<td>BH25</td>
<td>1.55 2.30</td>
<td>1.368 2.25</td>
<td>0.182 0.05</td>
</tr>
<tr>
<td>BH06</td>
<td>BH07</td>
<td>- -</td>
<td>0.934 2.25</td>
<td>- -</td>
</tr>
<tr>
<td>BH07</td>
<td>BH06</td>
<td>- -</td>
<td>0.990 2.25</td>
<td>- -</td>
</tr>
<tr>
<td>BH10</td>
<td>BH11</td>
<td>0.55 2.35</td>
<td>0.458 2.25</td>
<td>0.092 0.10</td>
</tr>
<tr>
<td>BH11</td>
<td>BH10</td>
<td>0.55 2.25</td>
<td>0.573 2.25</td>
<td>0.023 0.00</td>
</tr>
</tbody>
</table>
Fig. 5.41: Inversion results with the gradient error scheme from the data at boreholes BH25 - BH03 (a), BH03 - BH25 (b), BH06 - BH07 (c), BH07 - BH06 (d), BH10 - BH11 (e), and BH11 - BH10 (f). White crosses indicate the minimum errors.
Fig. 5.42: Vertical radar profile acquired by surface GPR along a survey line from borehole BH07 to borehole BH06.

Fig. 5.43: Scene of the pipe installation.
Fig. 5.44: Velocity distributions computed by straight-ray tomography from the data for BH25 - BH03 (a), BH03 - BH25 (b), BH06 - BH07 (c), BH07 - BH06 (d), BH11 - BH10 (e), respectively. The calculation for the data BH10 - BH11 is divergent and the result cannot be archived.
5.3 Experimental Results

5.3.4 Air-filled cavity

Data for localization of an air-filled cavity were acquired in Korea, in 2000 (Zhou and Sato, 2004). The vertical section of the site is illustrated in Fig. 5.45. The site consists of a tunnel between two boreholes separated by 19.5 m, and the tunnel has a diameter of about 3 m.

Inversion of cross-hole fan data

For cross-hole fan measurements, the transmitter was placed in borehole B2 at depths of 70, 75, 80, 85, and 90 m, respectively, and the receiver was scanned in borehole B1 ranging from 70 to 90 m at every 0.125 m. The obtained radar profiles are shown in Fig. 5.46, and one can easily find anomalies due to the cavity at a depth of around 80 m. Fig. 5.47 shows the result of the inversion with the gradient error scheme. As summarized in Table 5.9, the data for the transmitter depths of 75, 80, and 85 m are used. In the result, the minimum error appears at a depth of 80 m and 4.5 m apart from borehole B1. It agrees well with the imaging result in Zhou and Sato (2004).

Fig. 5.45: Geometrical sketch of the site for field measurements in Korea, 2000.
Fig. 5.46: Radar profiles obtained by cross-hole fan measurements for the cavity. The transmitter is set at depths of 70 m (a), 75 m (b), 80 m (c), 85 m (d), and 90 m (e) in borehole B2.
Table 5.9: Parameters for inverting the data measured in Korea, 2000.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Used Tx depths</td>
<td>75, 80, 85 m</td>
</tr>
<tr>
<td>Used Rx depths</td>
<td>73-83 m (Tx at 75 m), 73-85 m (Tx at 80 m), 73-84 m (Tx at 85 m)</td>
</tr>
<tr>
<td>Calculation step</td>
<td>0.1 m</td>
</tr>
<tr>
<td>Assumed permittivity</td>
<td>$\varepsilon_0$</td>
</tr>
<tr>
<td>Arrival time picking</td>
<td>Threshold: 30 % (Tx at 75, 80 m), 40 % (Tx at 85 m)</td>
</tr>
</tbody>
</table>

Fig. 5.47: Result of the inversion for the cavity with the gradient error scheme. The white cross indicates the minimum error at a depth of 80 m and 4.5 m away from B1.
5.4 Performance Evaluation

5.4.1 Resolution

The resolution of the inversion is roughly estimated. In a real measurement, the data are digitized by a time increment \( \Delta t \). A change within the time increment cannot be recognized, thus the minimum change of a traveltime must be \( \Delta t \). The propagation path length within the time resolution can be represented as

\[
\Delta L = \frac{\Delta t}{\sqrt{\varepsilon}}.
\]  

(5.14)

where \( \varepsilon \) is the permittivity of the surrounding medium. Since the inversion deals with the arrival time curves, which are also digitized, the resolution along the propagation path is \( \Delta L \). Here we consider the simplest configuration of a cross-hole fan measurement, which has antennas, homogeneous medium, and no object between the boreholes as shown in Fig. 5.48. The propagation path length can be given by

\[
L = \frac{d}{\cos \alpha}
\]  

(5.15)

where \( \alpha \) is a ray angle, and \( d \) is a borehole separation. According from Eqs. (5.14) and (5.15), \( z \) and \( x \) components of \( \Delta L \) can be represented as

![Fig. 5.48: Straight ray path between transmitter and receiver.](image-url)
\[ \Delta L_z = \Delta L \sin \alpha = \frac{\Delta t}{\sqrt{\varepsilon}} \sin \alpha , \quad (5.16a) \]
\[ \Delta L_x = \Delta L \cos \alpha = \frac{\Delta t}{\sqrt{\varepsilon}} \cos \alpha . \quad (5.16b) \]

In the inversion, the distributions of errors or similarities are summed, and each distribution has a different ray angle for a particular position. The resolution given by Eq. (5.16) is variable by the ray angle \( \alpha \), and that of the resultant distribution depends on the highest resolution. The resolution \( \Delta z \) and \( \Delta x \) might be given by the maximum \( \alpha \).

\[ \Delta z = \frac{\Delta t}{\sqrt{\varepsilon}} \sin \alpha_{\text{max}} = \frac{\Delta t}{\sqrt{\varepsilon}} \sin \left( \tan^{-1} \frac{dz_{\text{max}}}{d} \right) \quad (5.17a) \]
\[ \Delta x = \frac{\Delta t}{\sqrt{\varepsilon}} \cos \alpha_{\text{max}} = \frac{\Delta t}{\sqrt{\varepsilon}} \cos \left( \tan^{-1} \frac{dz_{\text{max}}}{d} \right) \quad (5.17b) \]

Where \( dz \) denotes a depth difference of a transmitter and a receiver. Note that the ray angle \( \alpha \) and depth difference \( dz \) in this discussion are defined by the path, which includes the target. It means that the resolution is also variable with the object location. The derivation does not consider the presence of the object, and the resolution must depend on the dimension. Thus, Eq. (5.17) is a rough estimation, and the actual resolution would be worth.

Eq. (5.17) shows that the resolution does not depend on density of the data acquisition, i.e., number of data. If the calculation is done with a very small number of data sets, the resultant distribution hardly has a region clearly localized. Whereas, the inversion determines the location of an object where the minimum/maximum value is located, and the value can be selected even if the distribution is relatively flat. This is different from conventional techniques, and Eq. (5.17) indicates that the inversion can work even with very sparse data.

The resolution for the measured data can be estimated as shown in Table 5.10.
5.4.2 Computational costs

The computation time is compared with that of traveltime tomography. The straight-ray tomography employs an *algebraic reconstruction technique* (ART) based algorithm (e.g., Dines and Lytle, 1979; Peterson *et al.*, 1985). The iteration is terminated when the differences between current and previous maximum prediction errors are within 0.001 ns. The comparison of the computation times between the new inversion and tomography are shown in Fig. 5.49 for the synthetic data and the experimental data in Sendai. The CPU times are measured on a PC with a CPU of Pentium 4 2.8 GHz and RAM of 2 GB. The black bars show the times taken for the inversion with the gradient error scheme, while the white bars show those for the tomography. The inversion of synthetic data takes 17% of the time needed for the tomography. In particular, the inversion is extremely faster than tomography in the cases of measured data; it is only less than 0.4%. Since the measured data is noisy, the tomography calculation takes much longer time to satisfy the termination criterion. Thus, the inversion technique is more efficient to be applied to noisy data. The computation times for the measured data in Amagasaki are shown in Fig. 5.50.

Table 5.10: Estimated resolutions for the measured data.

<table>
<thead>
<tr>
<th>Data</th>
<th>Permittivity</th>
<th>Maximum ray angle [degree]</th>
<th>Time resolution [ns]</th>
<th>Estimated resolution [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sendai</td>
<td>$20 \varepsilon_0$</td>
<td>26.57</td>
<td>1.24</td>
<td>3.7</td>
</tr>
<tr>
<td>Amagasaki, BH25-BH30</td>
<td>$5 \varepsilon_0$</td>
<td>49.40</td>
<td>1.24</td>
<td>12.6</td>
</tr>
<tr>
<td></td>
<td>$5 \varepsilon_0$</td>
<td>51.34</td>
<td>1.24</td>
<td>13.0</td>
</tr>
<tr>
<td></td>
<td>$5 \varepsilon_0$</td>
<td>68.20</td>
<td>1.24</td>
<td>6.2</td>
</tr>
<tr>
<td>Korea</td>
<td>$5 \varepsilon_0$</td>
<td>33.69</td>
<td>1.24</td>
<td>9.2</td>
</tr>
</tbody>
</table>


Fig. 5.49: CPU time for calculating for the inversion and traveltime tomography. Times are measured on a PC with Pentium 4 2.8 GHz CPU and 2 GB RAM.

Fig. 5.50: CPU time for calculating the inversion and traveltime tomography of the measured data in Amagasaki. Tomography of the data for BH10-BH11 is diverged.
5.5 An Application Example to Non-Cylindrical Object

5.5.1 Inversion for layered structure

In above sections, the inversion is demonstrated for cylindrical objects like pipes and tunnels. This section shows an example of the application to a non-cylindrical object. The parameter to be estimated is a vertical location of a layering boundary. As examined by Clement and Knoll (2000), it is difficult to obtain very clear images of the layered structure by ray tomography. Although it can determine a boundary depth, the image is smeared by artifacts and is confusing.

For demonstrating the ability of the inversion to estimate a depth of the layering interface by the new inversion, three-dimensional FDTD is used to generate the synthetic data. The interface of the two horizontal layers is set at a depth of 12 m as shown in Fig. 5.51. The two layers have different permittivity and conductivity, $20 \varepsilon_0$ and 0.01 S/m for the upper layer, and $30 \varepsilon_0$ and 0.05 S/m for the lower layer. Other parameters for the FDTD are listed in Table 5.11. The obtained data are shown in Fig. 5.52.

In this example, it is assumed that the permittivity and conductivity of the two layers are known, and the boundary is not dipping. The unknown is only the vertical location of the boundary.
5.5 An Application Example to Non-Cylindrical Object

Fig: 5.51: Cross-section of the FDTD model. The layer interface is modeled at a depth of 12 m.

Table 5.11: Parameters for the FDTD calculation with horizontal layers.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell size</td>
<td>$0.1 \text{ m} \times 0.1 \text{ m} \times 0.1 \text{ m}$</td>
</tr>
<tr>
<td>Number of cells</td>
<td>$100 \times 100 \times 80$</td>
</tr>
<tr>
<td>Time step</td>
<td>$0.19258078 \text{ ns}$</td>
</tr>
<tr>
<td>Number of time steps</td>
<td>1500</td>
</tr>
<tr>
<td>Boundary condition</td>
<td>10 layers PML</td>
</tr>
<tr>
<td>Excitation pulse</td>
<td>2nd derivative Gaussian pulse</td>
</tr>
<tr>
<td>Transmitter depths</td>
<td>11-13 m, 0.5 m step</td>
</tr>
<tr>
<td>Receiver depths</td>
<td>10-14 m, 0.1 m step</td>
</tr>
</tbody>
</table>
Fig. 5.52: Synthetic radar profiles generated for the model shown in Fig. 5.51. The transmitter is set at depths of 11.0 m (a), 11.5 m (b), 12.0 m (c), 12.5 m (d), and 13.0 m (e).
5.5.2 Forward modeling

By geometrical calculation similar to the cases of the pipe and cavity, arrival times cannot be obtained in the case of a non-cylindrical objects, because the model possesses shadow regions and there is an area that no ray can reach if only geometrical optics (Snell’s law) is considered. Hence, FDTD calculations are employed to obtain the first arrival times. To save the computational costs, a two-dimensional TE FDTD method without absorbing boundary condition (ABC) is implemented, and the calculations are terminated when the first arrivals are observed. The parameters for the forward modeling by means of 2D TE FDTD are summarized in Table 5.12.

5.5.3 Inversion result

The gradient error scheme is used in the inversion. Errors for a transmitter are shown in Fig. 5.53. As described above, the unknown is only the depth of the layer interface, thus the errors are distributed along the depth. Fig. 5.54 shows the summed error. The error has a minimum at a depth of 12 m, and the depth of the layering boundary can be estimated. It shows that the inversion can work not only for a cylindrical object but also for a planar object. If the inversion is performed also for permittivities and the dipping, those parameters would be estimable. This shows the potential of the technique to be applied to many targets.

Table 5.12: Parameters for the 2D TE FDTD calculation for the forward modeling.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell size</td>
<td>0.1 m×0.1 m</td>
</tr>
<tr>
<td>Number of cells</td>
<td>70 (z)×40 (x)</td>
</tr>
<tr>
<td>Time step</td>
<td>0.1 ns</td>
</tr>
<tr>
<td>Boundary condition</td>
<td>-</td>
</tr>
<tr>
<td>Excitation pulse</td>
<td>2nd derivative Gaussian pulse</td>
</tr>
</tbody>
</table>
Fig. 5.53: Error for transmitters at depths of 11.0 m (a), 11.5 m (b), 12.0 m (c), 12.5 m (d), and 13.0 m (e). Lower error indicates higher probability of the layer interface.
Fig. 5.54: Total error. The lowest error is at a depth of 12 m.